Critical Divergence in Holographic Foundations: Conflict with the French School, Ill-posed Relation with Negative Effective Mass, and the Higgs

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January 13, 2026

Abstract

Although holography successfully models negative mass-squared instabilities and symmetry breaking, it does not presently capture negative effective mass as a band-structure phenomenon, limiting its ability to establish a concrete connection between 2D planes and the Higgs field.We present a new but constructive critique of top-down cosmology and holographic frameworks (in particular the Hawking–Hertog approach), emphasizing precise conditions under which contour deformations and complex saddle prescriptions can break standard spectral/self-adjointness arguments. We add a focused review of the Breitenlohner–Freedman (BF) stability condition in AdS, a lemma and toy example that make the self-adjointness concern precise, and an expanded account of how the spectral action (Connes–Chamseddine) produces quadratic (mass) terms for internal scalar degrees of freedom (with this we mean the Higgs), with explicit reference to the heat-kernel expansion and where the sign and magnitude enter. Tachyonic bulk masses are compatible with AdS holography provided the Breitenlohner–Freedman bound is satisfied. However, whether such modes contribute physically in a given top-down cosmological construction depends on the fluctuation spectrum around the chosen semiclassical saddle. Consequently, claims about the retention or suppression of Higgs-like tachyonic sectors must in our view, always be justified by an explicit saddle-dependent mode analysis. We argue Hawking-Hertog their work is useful but incomplete.

1 Introduction

The holographic principle, particularly as realized in the AdS/CFT correspondence, provides a powerful toolkit for quantum gravity. The top-down cosmology proposed by Hawking and Hertog uses a semiclassical approximation of a Wave Function of the Universe $\Psi[h_{ij}, \phi]$ defined by a Euclidean path integral over histories ending on a spacelike boundary Σ:

$$\Psi[h_{ij}, \phi] = \int_{\mathcal{C}} \mathcal{D}g_{\mu\nu}\, \mathcal{D}\Phi \; e^{-S_E[g,\Phi]}. \tag{1}$$

This innovative approach legitimately relies on complex saddles and contour deformations. As observed in the literature, certain contour prescriptions can lead to mathematical and physical inconsistencies unless handled carefully (see [1, 2]).

In what follows we formalize a few critical errors in the current Holography theorem, and provide concrete steps to make the theory more solid: (i) precise statement of the BF bound and its relevance to tachyonic modes in AdS, (ii) a lemma on self-adjointness and a 1D toy example showing how complexified metrics (or complexified coefficients in a differential operator) can violate symmetry/self-adjointness assumptions, and (iii) an explicit heat-kernel expansion for the spectral action showing how quadratic Higgs-like terms arise and why their sign/coefficient depend on the cutoff and the moments of the test function used in the spectral action.

2 It's the scalar! Breitenlohner–Freedman bound and tachyonic bulk masses

Scalar fields in AdS do *not* require $m^2 \geq 0$ for stability. The correct stability condition in an $d+1$-dimensional AdS background is the Breitenlohner–Freedman (BF) bound. For an AdS radius L and boundary spacetime dimension d the relation between bulk mass and conformal dimension is

$$\Delta(\Delta - d) = m^2 L^2, \tag{2}$$

and the BF bound states

$$m^2 \geq m_{\mathrm{BF}}^2 = -\frac{d^2}{4L^2}. \tag{3}$$

Thus negative m^2 values are allowed (and common in holographic models) so long as (3) holds; when m^2 violates the BF bound, linearized perturbations are unstable and no unitary CFT dual exists in the usual sense [4, 5].

An important corollary (used widely in holographic renormalization) is the existence of a window

$$m_{\mathrm{BF}}^2 \leq m^2 < m_{\mathrm{BF}}^2 + 1,$$

in which two different quantizations (standard and alternate) are possible and correspond to different operator dimensions on the boundary; this is a technical route by which "tachyonic" bulk masses can nevertheless produce controlled boundary physics.

3 Contour deformations, complex saddles, and spectral/self-adjointness

Let us consider, the analytic continuation of the metric,

$$g_{\mu\nu} \mapsto g_{\mu\nu}^{\mathbb{C}},$$

could potentially break the spectral reality properties of operators like the (Euclidean) Dirac operator $\not{D}$ or fluctuation operators derived from the quadratic expansion of S_E.

Lemma 1 (Failure of self-adjointness under complex deformation – sketch)**.** *Let $\mathcal{D}$ be a formally self-adjoint elliptic operator (e.g. the Dirac operator or a Laplace-type operator) on a compact Riemannian manifold (M, g) with domain $C_c^\infty(M)$. If the metric is analytically continued so that the coefficients in $\mathcal{D}$ acquire nonzero imaginary parts in an open region (so the operator is no longer real-coefficient), then $\mathcal{D}$ need not be symmetric with respect to the original L^2 inner product; hence essential self-adjointness can fail, and the spectral theorem does not apply in the same way.*

Illustrative toy example (one-dimensional). Consider the one-dimensional Sturm–Liouville operator on $I = [0, 1]$ with Dirichlet boundary conditions,

$$L_\alpha = -\frac{d^2}{dx^2} + i\alpha, \qquad \alpha \in \mathbb{R}.$$

For smooth u, v vanishing at the boundary,

$$\langle u, L_\alpha v\rangle = \int_0^1 \overline{u}\left(-v'' + i\alpha v\right)\, dx,$$

while

$$\langle L_\alpha u, v\rangle = \int_0^1 \left(-\overline{u}'' - i\alpha\overline{u}\right) v\, dx.$$

Integrating the second-order parts by parts (boundary terms vanish), one finds

$$\langle u, L_\alpha v\rangle - \langle L_\alpha u, v\rangle = 2i\alpha \int_0^1 \overline{u} v \, dx,$$

which is nonzero for generic u, v whenever $\alpha \neq 0$. Thus L_α is not symmetric for $\alpha \neq 0$ with respect to the standard L^2 inner product, so it cannot be essentially self-adjoint on the original domain. Hence complex coefficients (coming, in the gravitational context, from complexified metric components along a chosen contour) can destroy the usual spectral properties relied upon in proven constructions. □

Remarks. The above does not mean complex saddles are always illegitimate. It means that any argument relying on spectral reality, positivity, or self-adjointness must explicitly track the domain, inner product, and contour choice; one cannot simply assume operator spectra remain purely real after complexification. The Euclidean no-boundary path integral is not invariant under arbitrary contour deformations in the complexified metric space. Explicit counterexamples show that particular contour choices can lead to divergent fluctuation spectra or loss of semiclassical control, emphasizing that contour selection constitutes a nontrivial part of the definition of the theory. [2, 3].

4 What Hertog and Hawking overlooked: The Higgs sector and Non-Commutative Geometry (spectral action)

In theory holographic derivations "miss" a tachyonic (negative mass-squared) sector which is essential for spontaneous symmetry breaking. However this notion should be somewhat nuanced:

- In AdS holography, negative bulk masses are allowed down to the BF bound; hence the mere presence of a negative mass is not in contradiction with holography.
- A structural difference arises describing the Higgs as an ordinary bulk scalar in a continuous geometry or deriving it as an inner fluctuation of a discrete/noncommutative internal space, as in the Connes–Chamseddine spectral action approach. The latter organizes Higgs-like terms as part of the Dirac operator fluctuations and produces quadratic and quartic terms after expansion. Hence you could say the theory has potential, however the current state of the theory is inherently flawed. Perhaps we can correct partition function.

Let us expand a bit further why the theory is flawed, covering spectral action and heat kernel expansion. Finally, we will cover partition function.

4.1 Spectral action and heat kernel expansion

The spectral action principle (Connes–Chamseddine) is

$$S_{\text{spectral}} = \langle \psi, \mathcal{D}\psi\rangle + \operatorname{Tr}\big(f(\mathcal{D}/\Lambda)\big),$$

where $\mathcal{D}$ is the (fluctuated) Dirac operator on the spectral triple, f is a positive even test function, and Λ is a high energy cutoff scale. The asymptotic expansion (heat-kernel style) for large Λ reads schematically (for a 4-dimensional manifold)

$$\operatorname{Tr}\big(f(\mathcal{D}/\Lambda)\big) \sim \Lambda^4 f_4\, a_0(\mathcal{D}^2) + \Lambda^2 f_2\, a_2(\mathcal{D}^2) + f_0\, a_4(\mathcal{D}^2) + \mathcal{O}(\Lambda^{-2}), \tag{4}$$

where the coefficients f_k are the momenta of f,

$$f_k := \int_0^\infty u^{k/2-1} f(u)\, du,$$

and the a_{2n} are the standard heat-kernel (Seeley–DeWitt) coefficients built from local curvature invariants and internal gauge / scalar terms (see [6–8]).

Important points for the Higgs mass:

- The a_2 coefficient multiplies $\Lambda^2 f_2$ and contains terms proportional to scalar-curvature-type invariants and mass-like bilinears for internal scalar fields that originate from inner fluctuations of $\mathcal{D}$. Thus a quadratic term in the Higgs field H, schematically $\sim c(\Lambda, f)\,|H|^2$, generically appears with coefficient proportional to $\Lambda^2 f_2$ times a geometric factor coming from a_2.

- The **sign** and size of this coefficient are *not universal* — they depend on the choice of f (via f_2), the detailed spectral triple (thus the geometry and discrete algebraic data), and the overall cutoff Λ. Hence spectral geometry gives a mechanism to produce a negative (tachyonic) quadratic term, but whether it is negative or positive must be computed in the chosen model and cannot be asserted a priori.

Thus the spectral action provides a controlled, geometric origin for a Higgs quadratic term; this is an alternative to treating the Higgs as a naive continuum bulk scalar and is precisely the sort of discrete internal structure the "French school" (Connes et al.) emphasizes [6, 7].

5 Proposed solution: A Corrected partition function and prescription

We can improve Hawking–Hertog's work as follows. One may need modified boundary terms to restore desired physical properties. A corrected partition function (schematic) that includes a non-minimal boundary coupling and a placeholder for the spectral corrections reads

$$\mathcal{Z}_{\text{mod}} = \int \mathcal{D}g\,\mathcal{D}\phi\;\exp\Big(-S_{HH}[g,\phi] + \int_{\partial\mathcal{M}} d^dx\sqrt{h}\,\xi\,\mathcal{R}[h]\,\phi^2 + S_{\text{spectral,bdry}}\Big), \tag{5}$$

where $S_{\text{spectral,bdry}}$ indicates boundary terms induced by spectral geometry or discrete internal structure. If one introduces any "ghost" operator $\hat{\mathcal{G}}$ on the boundary to mimic a negative-mass sector, it must be explicitly demonstrated how unitarity is preserved (e.g. BRST-type cancellation) or how the ghost is merely a bookkeeping device and not a physical negative-norm state.

6 Conclusion

Holography theory is useful, but will only survive providing we build further solutions upon the theorem. Negative effective mass should be further studied, and integrated within Holography theorem as our knowledge around the topic improves.

Recommendations for researchers: Compute the spectrum of the quadratic fluctuation operator and show whether negative eigenvalues are present and whether they are suppressed or enhanced by the chosen contour. Study BRST or constraint analysis that shows how unitarity is preserved (or will fail).

References

[1] S. W. Hawking and T. Hertog, "Populating the Landscape: A Top Down Approach," arXiv:hep-th/0602091 (Phys. Rev. D73 (2006) 123527).
`https://arxiv.org/abs/hep-th/0602091`

[2] J. Feldbrugge, J.-L. Lehners and N. Turok, "Inconsistencies of the New No-Boundary Proposal," arXiv:1805.01609 (2018).
https://arxiv.org/abs/1805.01609

[3] J. Diaz Dorronsoro et al., (see discussion and responses in the literature about circular-lapse contour proposals). Example discussions and replies are summarized in the literature cited in [2].

[4] P. Breitenlohner and D. Z. Freedman, "Stability in gauged extended supergravity," Annals Phys. **144** (1982) 249–281. (This is the standard reference for the BF bound.)
https://inspirehep.net/literature/12266

[5] S. S. Gubser and I. Mitra, "Some interesting violations of the Breitenlohner-Freedman bound," arXiv:hep-th/0108239.
https://arxiv.org/abs/hep-th/0108239

[6] A. Connes and A. H. Chamseddine, "The Spectral Action Principle," arXiv:hep-th/9606001.
https://arxiv.org/abs/hep-th/9606001

[7] A. H. Chamseddine and A. Connes, "The Spectral Action Principle", JHEP papers and follow-ups discuss the heat-kernel expansion and how SM terms appear; see also reviews and surveys on spectral action gravity and cosmology. Example survey: M. Marcolli, "Spectral action gravity and cosmological models," and related literature.
https://arxiv.org/abs/hep-th/9606001

[8] For spectral action boundary terms and explicit checks, see for instance: A. H. Chamseddine, A. Connes, M. Marcolli and follow-up literature; also "Quantum Gravity Boundary Terms from the Spectral Action," Phys. Rev. Lett. (example studies of boundary corrections).
https://doi.org/10.1103/PhysRevLett.99.071302